Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME: Solved by Smart Edu Hub

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MATHEMATICS **0580/42**

Paper 4 (Extended) **October/November 2018**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator
Tracing paper (optional)
Geometrical instruments

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.

1 **(a)** The Muller family are on holiday in New Zealand.

(i) They change some euros (€) and receive $1962 (New Zealand dollars).
The exchange rate is €1 = $1.635 .

Calculate the number of euros they change.

€1 = $1·635

€x = $ 1962

∴ 1·635x = 1962

x = 1962 ÷ 1·635

= 1200

€ 1200 [2]

(ii) The family spend 15% of their New Zealand dollars on a tour.

Calculate the number of dollars they have left.

$$1962 - \frac{15}{100}(1962) = 1667.7$$

$ 1667·7 [2]

(iii) The family visit two waterfalls, the Humboldt Falls and the Bridal Veil Falls.
The ratio of the heights Humbold' Falls : Bridal Veil Falls = 5 : 1.
The Humboldt Falls are 220 m higher than the Bridal Veil Falls.

Calculate the height of the Humboldt Falls.

Humbold falls : Bridal Veil falls

5 : 1

$$\frac{220}{(5-1)} + 220$$

$$= 55 + 220$$

$$= 275$$

275 m [2]

(b) **(i)** Water flows over the Browne Falls at a rate of 3680 litres per second.
After rain, this rate increases to 9752 litres per second.

Calculate the percentage increase in this rate.

$$\frac{9752-3680}{3680} \times 100 = 165$$

165 % [3]

(ii) After rain, water flows over the Sutherland Falls at a rate of 74 240 litres per second.
This is an increase of 45% on the rate before the rain.

Calculate the rate before the rain.

let the rate before rain = x

$$\therefore 74240 = x + \frac{45}{100}x$$

$$\therefore 7424000 = 145x$$

$$\therefore x = \frac{7424000}{145}$$

51200 litres/second [3]

2 (a) Solve $30 + 2x = 3(3 - 4x)$.

$30+2x = 3(3-4x)$

$30 + 2x = 9 - 12x$

$\therefore 2x + 12x = 9 - 30$

$14x = -21$

$x = -21 \div 14 = -1.5$

$x =$ −1·5 [3]

(b) Factorise $12ab^3 + 18a^3b^2$.

$12ab^3 + 18a^3b^2$

$= 6ab^2(2b + 3a^2)$

$6ab^2(2b+3a^2)$ [2]

(c) Simplify.

(i) $5a^3c^2 \times 2a^2c^7$

$5a^3c^2 \times 2a^2c^7$

$= 5 \times 2 \times a^3 \times a^2 \times c^2 \times c^9 = 10a^5c^9$

$10a^5c^9$ [2]

(ii) $\left(\frac{16a^8}{c^{12}}\right)^{\frac{3}{4}}$

$\frac{(16a^8)^{3/4}}{(c^{12})^{3/4}} = \frac{\left(\sqrt[4]{16a^8}\right)^3}{\left(\sqrt[4]{c^{12}}\right)^3} = \frac{(2a^2)^3}{(c^3)^3} = \frac{8a^6}{c^9}$

$\frac{8a^6}{c^9}$ [2]

(d) y is inversely proportional to the square of $(x + 2)$.
When $x = 3$, $y = 2$.

Find y when $x = 8$.

$y \propto \frac{1}{(x+2)^2}$

$\Rightarrow y = \frac{k}{(x+2)^2}$

When $x = 3$; $y = 2$

$\therefore 2 = \frac{k}{(3+2)^2} \Rightarrow k = 2(25) = 50$

$y = \frac{50}{(x+2)^2}$

$\therefore y = \frac{50}{(8+2)^2}$

$\therefore y = \frac{50}{(10)^2} = \frac{50}{100} = 0.5$

$y =$ 0·5 [3]

(e) Write as a single fraction in its simplest form.

$$\frac{5}{x-2} - \frac{x-5}{2}$$

$$\frac{5}{x-2} - \frac{x-5}{2}$$

$$\frac{2(5) - (x-2)(x-5)}{2(x-2)}$$

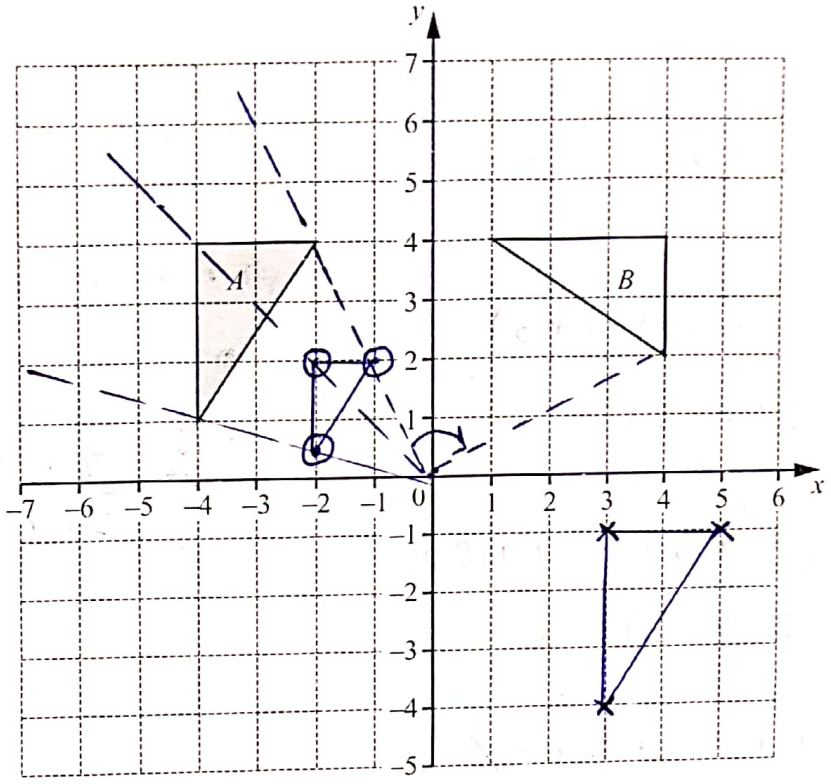
$$= \frac{10 - (x^2 - 5x - 2x + 10)}{2(x-2)}$$

$$= \frac{10 - x^2 + 5x + 2x - 10}{2(x-2)}$$

$$= \frac{7x - x^2}{2(x-2)}$$

$\frac{7x-x^2}{2(x-2)}$ ..... [3]

3



(a) Describe fully the **single** transformation that maps triangle A onto triangle B.

Rotation, 90° clockwise about origin

[3]

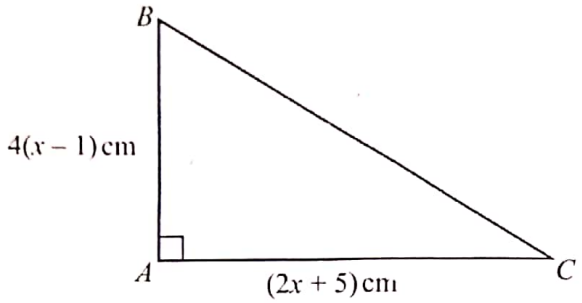
(b) On the grid, draw the image of

(i) triangle A after a reflection in the x-axis, [1]

(ii) triangle A after a translation by the vector $\begin{pmatrix} 7 \\ -5 \end{pmatrix}$, [2]

(iii) triangle A after the transformation represented by the matrix $\begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$. [3]

4 The diagram shows a right-angled triangle ABC.



NOT TO SCALE

The area of this triangle is $30\,cm^2$.

(a) Show that $2x^2 + 3x - 20 = 0$.

AREA OF THE TRIANGLE

$= \frac{1}{2} \times$ base $\times$ height

$\therefore 30 = \frac{1}{2} \times (2x+5) \times 4(x-1)$

$\therefore 60 = (2x+5)(4x-4)$

$= 2x(4x-4) + 5(4x-4)$

$60 = 8x^2 - 8x + 20x - 20$

$60 = 8x^2 + 12x - 20$

$\therefore 80 = 8x^2 + 12x$

$\therefore 8x^2 + 12x - 80 = 0$

$4(2x^2 + 3x - 20) = 0$

$2x^2 + 3x - 20 = 0$

[3]

(b) Use factorisation to solve the equation $2x^2 + 3x - 20 = 0$.

$2x^2 + 3x - 20 = 0$

$2x^2 + 8x - 5x - 20 = 0$

$2x(x+4) - 5(x+4) = 0$

$(2x-5)(x+4) = 0$

$2x - 5 = 0$ OR $x + 4 = 0$

$2x = 5$ $\quad x = -4$

$x = 2.5$

$x =$ 2·5 or $x =$ −4 [3]

(c) Calculate BC.

$BC = \sqrt{(4(x-1))^2 + (2x+5)^2}$

$= \sqrt{16(x^2 - 2x + 1) + (4x^2 + 20x + 25)}$

$= \sqrt{16x^2 - 32x + 16 + 4x^2 + 20x + 25}$

$= \sqrt{20x^2 - 12x + 41}$

; Subst $x = 2.5$ we get

$BC = 11.66 = 11.7$

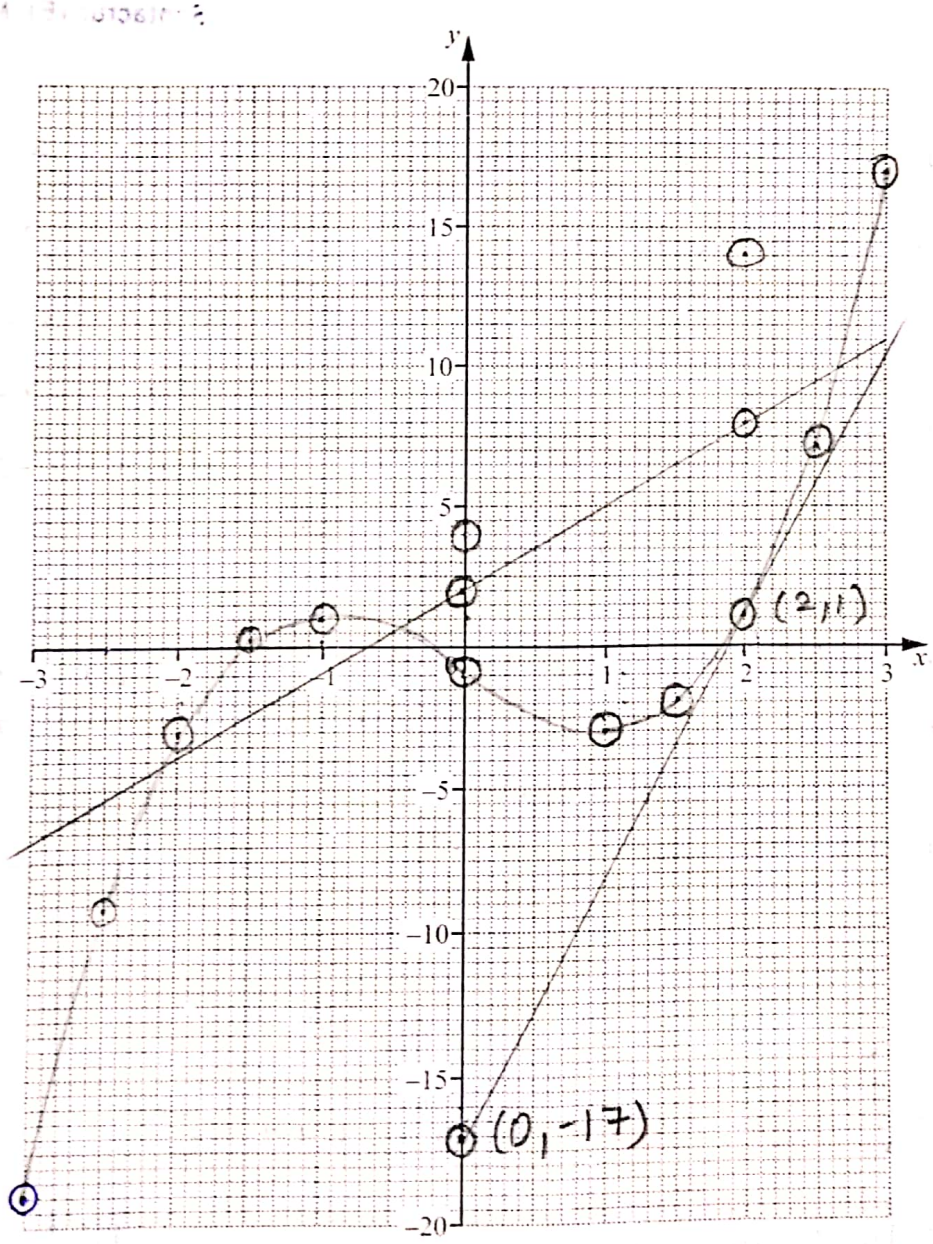
$BC =$ 11·7 cm [3]

5 The table shows some values of $y = x^3 - 3x - 1$.

x	−3	−2.5	−2	−1.5	−1	0	1	1.5	2	2.5	3
y	−19	−9.1	−3	0.1	1	−1	−3	−2.1	1	7.1	17

(a) Complete the table of values. [2]

(b) Draw the graph of $y = x^3 - 3x - 1$ for $-3 \leqslant x \leqslant 3$.



[4]

(c) A straight line through (0, –17) is a tangent to the graph of $y = x^3 - 3x - 1$.

(i) On the grid, draw this tangent. [1]

(ii) Find the co-ordinates of the point where the tangent meets your graph.

[(1·7 to 2·2), (–1 to 2·5)]
Accepted Range ↗

(..... 2 ..... , ... 1 .....) [1]

(iii) Find the equation of the tangent.
Give your answer in the form $y = mx + c$.

Equation = $y = mx + c$
$y = mx - 17$

$\therefore$ Equation is:
$y = 9x - 17$

Slope = $\frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{1 - (-17)}{2 - 0}$
$= \frac{1+17}{2}$
$= \frac{18}{2}$
$= 9$

$y =$ ..... 9x - 17 ..... [3]

(d) By drawing a suitable straight line on the grid, solve the equation $x^3 - 6x - 3 = 0$.

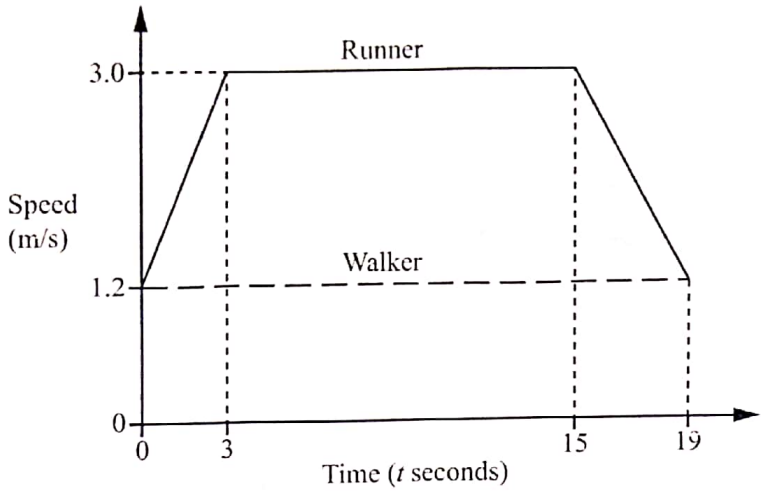
Draw $y = 3x + 2$ and check the intersection of both graphs

$y = x^3 - 3x - 1 = x^3 - 6x - 3$
$y = -3x + 6x - 1 + 3$
$y = 3x + 2$

Hence draw
$y = 3x + 2$

$x =$ ..... -2·1 ..... or $x =$ ..... -0·4 ..... or $x =$ ..... 2·7 ..... [4]

6 The diagram shows the speed–time graph for part of a journey for two people, a runner and a walker.



NOT TO SCALE

(a) Calculate the acceleration of the runner for the first 3 seconds.

Acceleration = Slope $= (y_2 - y_1) \div (x_2 - x_1)$

$= (3 - 1{\cdot}2) \div 3$

$= 0{\cdot}6$

0·6 ..... m s^{-2} [1]

(b) Calculate the total distance travelled by the runner in the 19 seconds.

Total distance = Area under the graph

$= \left[\frac{1}{2}(3) \times 1{\cdot}8\right] + [3 \times 1{\cdot}2] + [12 \times 3] + [4 \times 1{\cdot}2] + \left[\frac{1}{2} \times 4 \times 1{\cdot}8\right]$

$= 50{\cdot}7$

50·7 ..... m [3]

(c) The runner and the walker are travelling in the same direction along the same path.
When $t = 0$, the runner is 10 metres behind the walker.

Find how far the runner is ahead of the walker when $t = 19$.

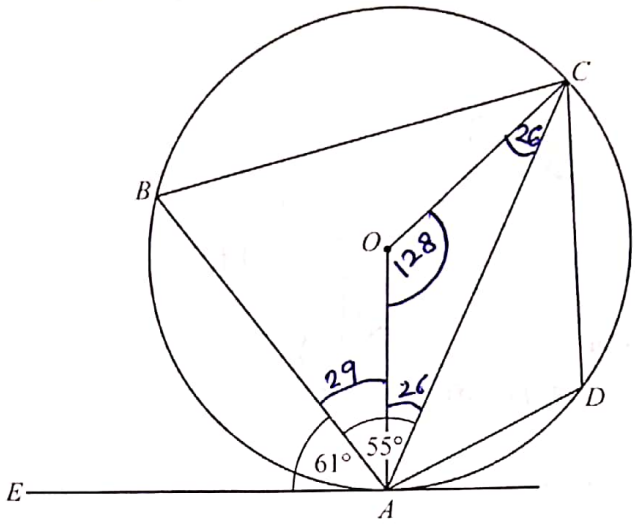
(Total distance of runner + walker) − (distance of the walker) − 10

$= 50{\cdot}7 - (1{\cdot}2 \times 19) - 10$

$= 17{\cdot}9$m

17·9 ..... m [3]

7



NOT TO SCALE

In the diagram, A, B, C and D lie on the circle, centre O.
EA is a tangent to the circle at A.
Angle $EAB = 61°$ and angle $BAC = 55°$.

(a) Find angle BAO.

$\angle BAO = 90 - 61 = 29^\circ$

Angle BAO = 29 [1]

(b) Find angle AOC.

$\angle AOC = 180 - 2(26) = 128$

Angle AOC = 128 [2]

(c) Find angle ABC.

$\angle ABC = \frac{1}{2}(AOC) = \frac{1}{2}(128) = 64^\circ$

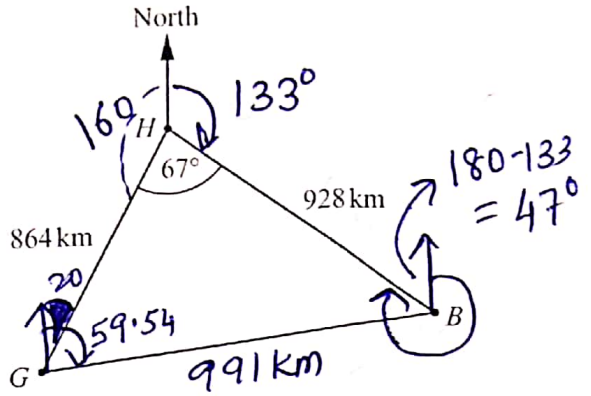
Angle ABC = 64 [1]

(d) Find angle CDA.

$\angle CDA = 180 - \angle ABC$
$= 180 - 64$
$= 116^\circ$

Angle CDA = 116 [1]

8 The diagram shows the positions of three cities, Geneva (G), Budapest (B) and Hamburg (H).



NOT TO SCALE

(a) A plane flies from Geneva to Hamburg.
The flight takes 2 hours 20 minutes.

Calculate the average speed in kilometres per hour.

Average Speed = Total distance ÷ Total time

= 864 ÷ 2·33 = 370·3

370 km/hr

370 km/h [2]

(b) Use the cosine rule to calculate the distance from Geneva to Budapest.

$GB^2 = 864^2 + 928^2 - 2(864)(928)\cos 67$

$\therefore GB = \sqrt{864^2 + 928^2 - 2(864)(928)\cos 67}$

$= 991$ km

991 km [4]

(c) The bearing of Budapest from Hamburg is 133°.

(i) Find the bearing of Hamburg from Budapest.

$= 360 - 47 = 313$

313 ..... [2]

(ii) Calculate the bearing of Budapest from Geneva.

$$\frac{\sin 67}{991} = \frac{\sin G}{928}$$

$$\Rightarrow \sin G = \frac{928 \times \sin 67}{991}$$

$$\Rightarrow G = \sin^{-1}\left(\frac{928 \times \sin 67}{991}\right)$$

$= 59.54$

$\therefore$ Required bearing

$= 59.54$

$+ 20.00$

$79.54 = 79.5$

79.5 ..... [4]

9 (a) The table shows the amount of time, T minutes, 120 people each spend in a supermarket one Saturday.

Midpt (CI) x	Time (T minutes)	Number of people
20	$10 < T \leqslant 30$	16
35	$30 < T \leqslant 40$	18
42·5	$40 < T \leqslant 45$	22
47·5	$45 < T \leqslant 50$	40
55	$50 < T \leqslant 60$	21
65	$60 < T \leqslant 70$	3

(i) Use the mid-points of the intervals to calculate an estimate of the mean.

Mean = $[(16 \times 20) + (18 \times 35) + (22 \times 42{\cdot}5) + (40 \times 47{\cdot}5) + (21 \times 55) + 3 \times 65)] \div 120$

$= 42{\cdot}8$

42·8 min [4]

(ii) Complete this histogram to show the information in the table.

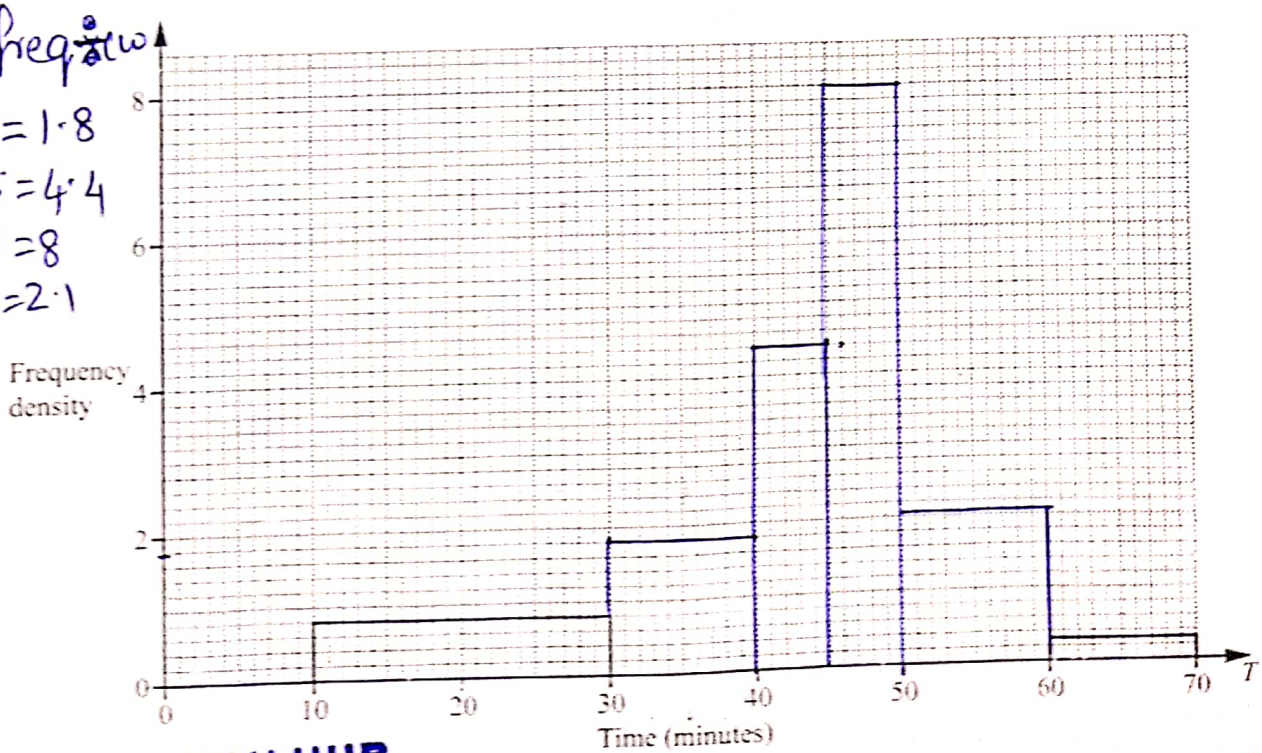
F·d = freq ÷ w

18 ÷ 10 = 1·8

22 ÷ 5 = 4·4

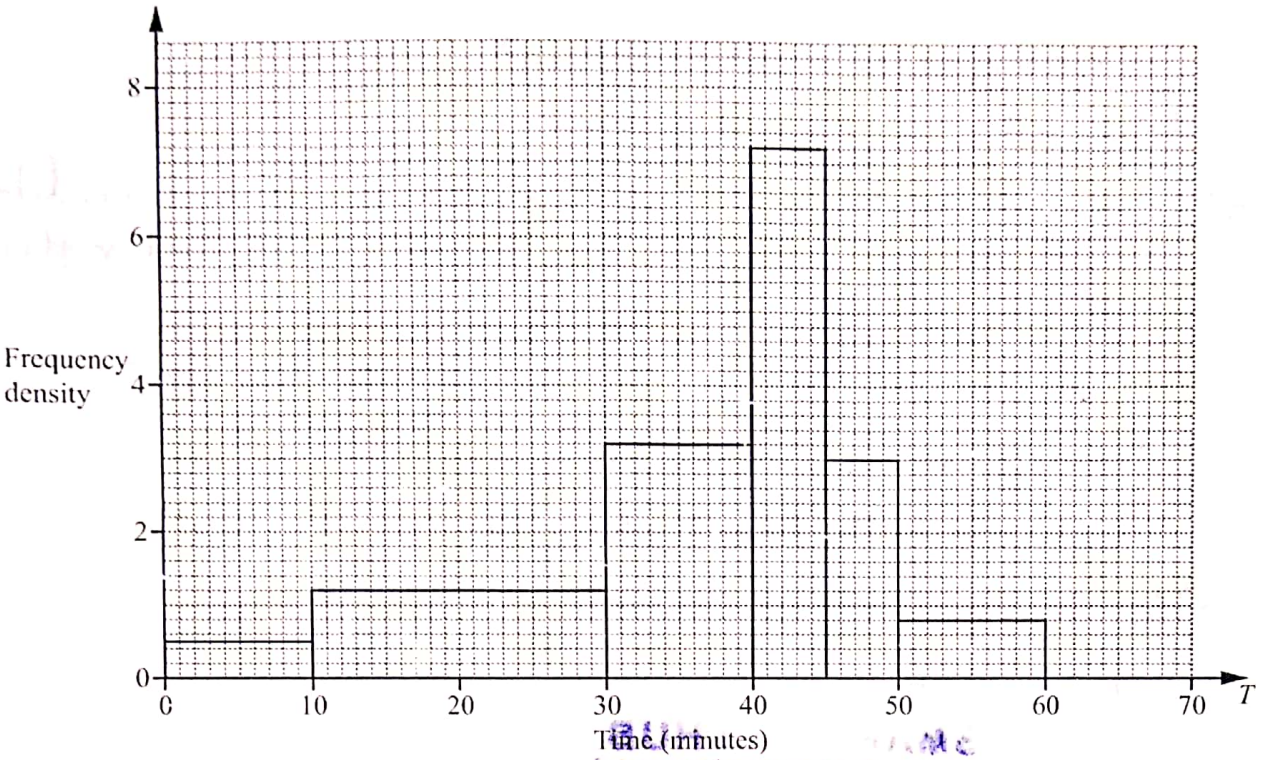
40 ÷ 5 = 8

21 ÷ 10 = 2·1



[4]

(b) This histogram shows the amount of time, T minutes, 120 people each spend in the supermarket one Wednesday.



Make a comment comparing the distributions of the times for the two days.

On an average, shoppers spend less time shopping on wednesday. [1]

10 (a) The lake behind a dam has an area of 55 hectares.
When the gates in the dam are open, water flows out at a rate of 75 000 litres per second.

(i) Show that 90 million litres of water flows out in 20 minutes.

55 hectares = 550 000 m^2

1 second = 75000 l of water

∴ 20 × 60 seconds ⇒ 75000 × 20 × 60 = 90 million litres of water flow

(ii) Beneath the surface, the lake has vertical sides.

Calculate the drop in the water level of the lake when the gates are open for 20 minutes.
Give your answer in centimetres.
[1 hectare = $10^4 m^2$, 1000 litres = $1 m^3$]

55 hectares = 550 000 m^2

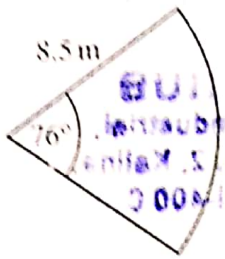
Volume = A(base) × height

900000 = 550000 × h

∴ h = 16.36

16.4 ..... cm [3]

(iii)



NOT TO SCALE

The cross-section of a gate is a sector of a circle with radius 8.5 m and angle 76°.

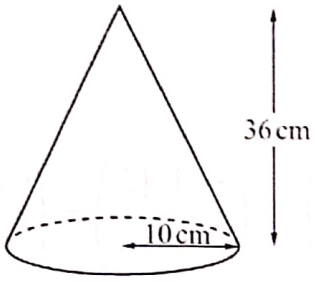
Calculate the perimeter of the sector.

Perimeter = $2(8.5) + \frac{76}{360} \times 2 \times \pi (8.5)$

= 28.3

28.3 ..... m [3]

(b)



NOT TO SCALE

A solid metal cone has radius 10 cm and height 36 cm.

(i) Calculate the volume of this cone.

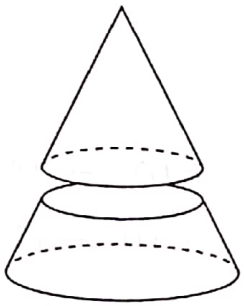
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Volume $= \frac{1}{3} \times \pi \times 10^2 \times 36$

$= 3770$

..... 3770 ..... cm^3 [2]

(ii) The cone is cut, parallel to its base, to give a smaller cone.



NOT TO SCALE

The volume of the smaller cone is half the volume of the original cone.
The smaller cone is melted down to make two different spheres.
The ratio of the radii of these two spheres is 1 : 2.

Calculate the radius of the smaller sphere.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

Volume of smaller cone $= \frac{1}{2}(3770) = 1885\ cm^3$

Let r = radius of smaller sphere
$\therefore 2r$ = radius of bigger sphere

$\therefore \frac{4}{3}\pi(r)^3 + \frac{4}{3}\pi(2r)^3 = 1885$

$\therefore \frac{4}{3}\pi r^3 + \frac{32}{3}\pi r^3 = 1885$

$36\pi r^3 = 1885 \times 3$

$\therefore r^3 = \frac{1885 \times 3}{36\pi}$

$r = \sqrt[3]{\frac{1885 \times 3}{36\pi}}$

$\therefore r = 3.68$

..... $\therefore r = 3.68$ cm [4]

11 (a) $\mathbf{a} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ $\mathbf{c} = \begin{pmatrix} 14 \\ 9 \end{pmatrix}$

(i) Find $3\mathbf{a} - 2\mathbf{b}$.

$3a-2b$

$= 3\begin{pmatrix} -3 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} -9 \\ 6 \end{pmatrix} - \begin{pmatrix} 10 \\ 8 \end{pmatrix} = \begin{pmatrix} -9-10 \\ 6-8 \end{pmatrix} = \begin{pmatrix} -19 \\ -2 \end{pmatrix}$

$\begin{pmatrix} -19 \\ -2 \end{pmatrix}$ [2]

(ii) Find $|\mathbf{a}|$.

$|a| = \sqrt{(-3)^2 + (2)^2}$

$|a| = \sqrt{9+4} = \sqrt{13} = 3.60$

3.61 [2]

(iii) $m\mathbf{a} + n\mathbf{b} = \mathbf{c}$

Write down two simultaneous equations and solve them to find the value of m and the value of n. Show all your working.

$ma + nb = c$

$2(-3m + 5n = 14) = -6m + 10n = 28$

$3(2m + 4n = 9) \quad 6m + 12n = 27$

$22n = 55$

$\therefore n = 55 \div 22 = 2.5$

$2m + 4n = 9$

$\therefore 2m + 4(2.5) = 9$

$2m + 10 = 9$

$2m = 9 - 10$

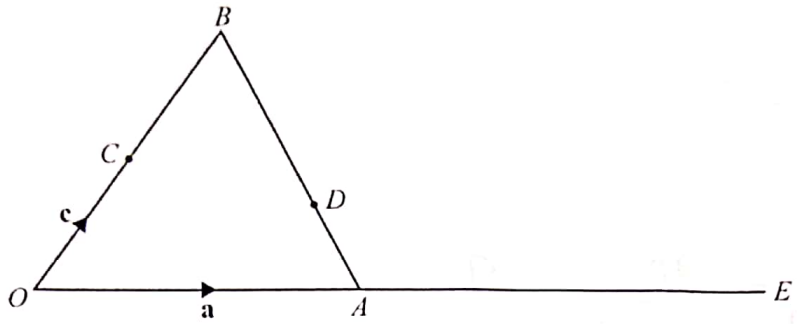
$2m = -1$

$m = -\frac{1}{2} = -0.5$

$m =$ −0.5

$n =$ 2.5 [5]

(b)



NOT TO SCALE

OAB is a triangle and C is the mid-point of OB.
D is on AB such that $AD : DB = 3 : 5$.
OAE is a straight line such that $OA : AE = 2 : 3$.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

(i) Find, in terms of **a** and **c**, in its simplest form,

(a) $\overrightarrow{AB}$,

$\overrightarrow{AB} = -a + 2c$

$\overrightarrow{AB} =$ $-a+2c$ [1]

(b) $\overrightarrow{AD}$,

$\overrightarrow{AD} = \frac{3}{8}(-a+2c) = -\frac{3a}{8} + \frac{3}{4}c$

$\overrightarrow{AD} =$ $-\frac{3}{8}a + \frac{3}{4}c$ [1]

(c) $\overrightarrow{CE}$,

$\overrightarrow{CE} = \frac{1}{2}(5a - 2c) = \frac{5}{2}a - c$

$\overrightarrow{CE} =$ $\frac{5}{2}a - c$ [1]

(d) $\overrightarrow{CD}$.

$\frac{1}{8}(5a - 2c) = \frac{5}{8}a - \frac{1}{4}c$

$\overrightarrow{CD} =$ $\frac{5}{8}a - \frac{1}{4}c$ [2]

(ii) $\overrightarrow{CE} = k\overrightarrow{CD}$

Find the value of k.

$\frac{5a}{2} - c = k\left(\frac{5}{8}a - \frac{1}{4}c\right)$

$\therefore k = 4.$

$k =$ 4 [1]

Question 12 is printed on the next page.

12 A box contains 20 packets of potato chips.

6 packets contain barbecue flavoured chips.
10 packets contain salt flavoured chips.
4 packets contain chicken flavoured chips.

(a) Maria takes two packets at random **without replacement.**

(i) Show that the probability that she takes two packets of salt flavoured chips is $\frac{9}{38}$.

$\frac{10}{20} \times \frac{9}{19} = \frac{90}{380} = \frac{9}{38}$

[2]

(ii) Find the probability that she takes two packets of different flavoured chips.

$1 - \frac{6}{20} \times \frac{5}{19} - \frac{10}{20} \times \frac{9}{19} - \frac{4}{20} \times \frac{3}{19}$

$= \frac{62}{95}$

$\frac{62}{95}$ [4]

(b) Maria takes three packets at random, **without replacement**, from the 20 packets.

Find the probability that she takes **at least two** packets of chicken flavoured chips.

Probability $= 1 - \left[3 \times \left(\frac{4}{20} \times \frac{16}{19} \times \frac{15}{18}\right) + \frac{16}{20} \times \frac{15}{19} \times \frac{14}{18}\right]$

$= \frac{5}{57}$

$\frac{5}{57}$ [3]