FUNCTIONS

1

(a) It is given that $f(x) = \frac{1}{2+x}$ for $x \neq -2, x \in \mathbb{R}$. [2]

(i) Find $f''(x)$.

(ii) Find $f^{-1}(x)$. [2]

(iii) Solve $f^2(x) = -1$. [3]

(b) The functions g, h and k are defined, for $x \in \mathbb{R}$, by

$$g(x) = \frac{1}{x + 5}, \quad x \neq -5,$$

$$h(x) = x^2 - 1,$$

$$k(x) = 2x + 1.$$

Express the following in terms of g, h and/or k.

(i) $\dfrac{1}{(x^2-1) + 5}$ [1]

(ii) $\dfrac{2}{x + 5} + 1$ [1]

(a) (i) $f'(x) = -(2+x)^{-2}$	**B1**	First **B1** may be implied by a correct answer for $f''(x)$
$f''(x) = 2(2+x)^{-3}$	**B1**	If done by quotient rule, allow unsimplified
(ii) $y = \frac{1}{2+x}$, $x = \frac{1}{y} - 2$	**M1**	**M1** for a valid attempt at the inverse
$f^{-1}(x) = \frac{1}{x} - 2$ or $\frac{1-2x}{x}$	**A1**	**A1** must be in correct form, allow $y = \ldots$
(iii) $f^2(x) = \left(\frac{1}{2+\frac{1}{2+x}}\right) = \frac{2+x}{5+2x}$	**M1**	**M1** for correct attempt at $f^2(x)$
	DM1	**DM1** for attempt at solution of $f^2(x) = -1$
Equating to -1 leads to $x = -\frac{7}{3}$ or -2.33	**A1**	**A1** for $x = -\frac{7}{3}$ or equivalent
(b) (i) gh (x) or gh	**B1**	**B1** for either form
(ii) kg (x) or kg	**B1**	**B1** for either form
	[9]	

2

(a) Functions f and g are defined, for $x \in \mathbb{R}$, by

$$f(x) = 3 - x,$$
$$g(x) = \frac{x}{x+2}, \quad \text{where } x \neq -2.$$

(i) Find $fg(x)$. [2]

(ii) Hence find the value of x for which $fg(x) = 10$. [2]

(b) A function h is defined, for $x \in \mathbb{R}$, by $h(x) = 4 + \ln x$, where $x > 1$.

(i) Find the range of h. [1]

(ii) Find the value of $h^{-1}(9)$. [2]

(iii) On the same axes, sketch the graphs of $y = h(x)$ and $y = h^{-1}(x)$. [3]

(a) **(i)** $\text{fg}(x) = \text{f}\left(\dfrac{x}{x+2}\right)$			
$= 3 - \dfrac{x}{x+2}$	A1	[2]	
(ii) $3 - \dfrac{x}{x+2} = 10$	DM1		DM1 for dealing with fractions sensibly
leading to $x = -1.75$	A1	[2]	
(b) **(i)** $\text{h}(x) > 4$	B1	[1]	
(ii) $\text{h}^{-1}(x) = \text{e}^{x-4}$	M1		M1 for attempting to obtain inverse function
$\text{h}^{-1}(9) = \text{e}^5 \quad (\approx 148)$	A1	[2]	
or $4 + \ln x = 9$,			
leading to $x = \text{e}^5$			
(iii) correct graphs	B1		B1 for each curve
	B1		
	B1	[3]	B1 for idea of symmetry